Mark Scheme 4725
June 2005

1.	$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated
	$2\Sigma r = n(n+1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
	$n(2n^2 + 4n + 3)$	M1	6	Correct algebraic processes including factorisation and simplification
		A1	**6**	Obtain given answer correctly
2.	(i) $A^2 = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$	M1		Attempt to find A^2, 2 elements correct
		A1		All elements correct
	$4A = \begin{pmatrix} 4 & 8 \\ 4 & 12 \end{pmatrix}$	M1		Use correct matrix 4**A**
	$\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$	A1	4	Obtain given answer correctly
	(ii)	M1		Multiply answer to (i) by $\mathbf{A}^{-1}$ or obtain $\mathbf{A}^{-1}$ or factorise $\mathbf{A}^2 - 4\mathbf{A}$
	$\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$	A1	2	Obtain given answer correctly
			6	
3.	(i) 22 – 2i	B1B1	2	Correct real and imaginary parts
	(ii) $z^* = 2 - 3i$	B1		Correct conjugate seen or implied
	5 – 14i	B1B1	3	Correct real and imaginary parts
	(iii) $\frac{4}{17} + \frac{1}{17}i$	M1		Attempt to use w^*
		A1	2	Obtain correct answer in any form
			7	

4.	$x^2 - y^2 = 21$ and $xy = -10$ $\pm(5 - 2i)$	M1 A1A1 M1 M1 A1	 6 **6**	Attempt to equate real and imaginary parts of $(x + iy)^2$ and 21 –20i Obtain each result Eliminate to obtain a quadratic in x^2 or y^2 Solve to obtain $x = (\pm)\,5$ or $y = (\pm)\,2$ Obtain correct answers as complex numbers
5.	(i) $\dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$ $\dfrac{1}{(r+1)(r+2)}$	M1 A1	 2	Show correct process for subtracting fractions Obtain given answer correctly
	(ii) *EITHER* $\frac{2}{3} - \frac{1}{2} + \frac{3}{4} - \frac{2}{3} \ldots \frac{n+1}{n+2} - \frac{n}{n+1}$ $\frac{n+1}{n+2} - \frac{1}{2}$	M1 A1 M1 A1	 4	Express terms as differences using (i) At least first two and last term correct Show or imply that pairs of terms cancel Obtain correct answer in any form
	OR	M2 A1A1		State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$ Each term correct
	(iii) $\frac{1}{2}$	B1 ft	1 **7**	Obtain value from their sum to *n* terms
6.	(i) Circle Centre (0, 2) Radius 2 Straight line Through origin with positive slope	B1 B1 B1 B1 B1	 5	Sketch(s) showing correct features, each mark independent
	(ii) 0 or 0 +0i and 2 + 2i	B1ftB1f t	2 **7**	Obtain intersections as complex numbers
8.	(a) (i) $\alpha + \beta = 2 \quad \alpha\beta = 4$	B1B1	2	Values stated
	(ii) *EITHER* $\alpha^2 + \beta^2 = -4$	M1 A1	 2	Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ Obtain given answer correctly
	OR	M1 A1		Find numeric values of roots, square and add Obtain given answer correctly
	(iii) $x^2 + 4x + 16 = 0$	B1		State or use $\alpha^2\beta^2 = 16$

		M1 A1	 3	Or use substitution $u = x^2$ Write down a quadratic equation of correct form or rearrange and square Obtain $x^2 + 4x + 16 = 0$
	(b) (i) $p = 2$	M1 A1	 2	Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$ Obtain $p = 2$
	(ii) $a = 44$	M1 A1ft	 2	Attempt to find $\Sigma\alpha\beta$ numerically or in terms of p or substitute their 2, 4 or 6 in equation Obtain $11p^2$
			11	
9.	(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1B1	2	Each column correct
	(ii) Shear, e.g. (0,1) transforms to (3,1)	B1B1	2	One example or sensible explanation
	(iii) $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$	M1 A1	 2	Attempt to find **DC** (not **CD**) Obtain given answer
	(iv)	B1		Explicit check for n = 1or n = 2
	$\mathbf{M}^k = \begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$.	M1		Induction hypothesis that result is true for $\mathbf{M}^k$
		M1		Attempt to multiply $\mathbf{MM}^k$ or vice versa
	$\begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$.	A1 A1		Element $3(2^{k+1} - 1)$ derived correctly All other elements correct
		A1	6	Explicit statement of induction conclusion
			12	